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J.M. ANTHONISSE, B.J. LAGEWEG & B. VAN RIJ

TIMETABLES FOR DENTAL STUDENTS

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Timetables for dental students*)

by

J.M. Anthonisse, B.J. Lageweg & B. van Rij**)

ABSTRACT

We consider a timetabling problem for the practical work which is to be performed by dental students. At the beginning of the academic year each student receives an individual timetable, indicating his activities for the entire year. The timetables have to observe several types of requirements, such as capacity, frequency and precedence constraints. Moreover the capacity of one particular practicum should be equally distributed over the students, leaving as little spare capacity as possible. The timetabling problem has been decomposed into a series of sequencing problems which are solved by heuristic methods. A set of FORTRAN programs has been written to store, retrieve, modify and print the individual timetables and related surveys. This approach allows for a large degree of flexibility in case of changes of the courses. The system has been operative for two years at the University of Amsterdam.

KEY WORDS & PHRASES: *timetabling, sequencing*

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*) This report will be submitted for publication elsewhere.

**) Government service for land and water use, Utrecht

1. INTRODUCTION

1.1. Problem description

We consider the timetabling problem for dental students at the University of Amsterdam in the academic year 1978/1979. The fourth, fifth and sixth year courses consist of the *practicums* listed in Table 1.

TABLE 1. LIST OF PRACTICUMS 1978/1979

Fourth year	Fifth year	Sixth year
patients' treatment	patients' treatment	patients' treatment
endodontics	surveying	oral surgery
pathological anatomy	dental technics	periodontics
general activities	prosthodontics	periodontics (patients)
orthodontics	oral surgery	dental technics
pharmacology	policlinic	
microbiology	periodontics	
prosthodontics	periodontics (pig's jaw)	
oral surgery	periodontics (patients)	
dental materials		
pedodontics		
roentgen diagnostics		
periodontics		
oral diagnosis		
oral hygiene		

The academic year starts on Monday 18th September 1978 and ends on Friday 7th September 1979. At the end of June nearly all courses are completed; merely a few practicums, among which patients' treatment, are continued during the summer. Immediately after the Christmas and Easter holidays general examinations are scheduled during which no other activities can take place. Moreover on public holidays such as the Queen's Anniversary and Ascension Day all activities are canceled.

Each activity can be split up into half days, i.e. mornings and afternoons. A week is subdivided into 10 half days or *segments*. A year consists of 52 weeks, each with 10 segments, hence altogether 520 *half days*.

For each practicum is known on which half days during the year it can be given and how many half days a student has to participate. The capacity on a given half day is mostly not so large as to allow all students, so the students have to be divided into fixed or varying groups. Participation in some practicums is only possible after completion of another one.

One part of each year's course, *patients' treatment*, has different characteristics from the other practicums. Each student is assigned to a dental instructor and is only allowed to treat patients during the segments of the week that his instructor is present. Each dental instructor is able to supervise at most eight students at the same time, also because of the layout of the clinic concerned, whereas the number of students allocated to him is in general larger than eight. The number of patients' treatments of each student has not been fixed a priori, but should be as large as possible, leaving as little unused capacity in the clinic as possible. Moreover the clinic's capacity should be fairly distributed among the students, i.e. each student should get allocated about the same number of half days of patients' treatment.

Hence we have to construct timetables which observe the constraints for all practicums inclusive patients' treatment, minimizing the unused capacity of patients' treatment.

At the moment of constructing the timetables it is known which, and therefore how many students are going to take part in the various activities. However, in September part of the students is conditionally admitted to the next year of their study. These so called *probationers* participate in all practicums of the new year, except for patients' treatment, in which subject the practicum of previous year has to be continued up to Christmas. Nearly all probationers are expected to be atmitted at that time to the next year of their study; so starting at January they have to be allotted for the patients' treatment concerned. The other probationers, not admitted to the next part, should continue in January with the same patients' treatment and besides should participate to a few practicums of the previous year.

TABLE 2. NUMBER OF STUDENTS

Time of allotment	Year					
	4		5		6	
	Probationer		Probationer		Probationer	
	no (4)	yes (4P)	no (5)	yes (5P)	no (6)	yes (6P)
September	88	28	74	23	73	-
January	88	6	96	7	89	-

The numbers of students are given in Table 2. From September on the sixth year's practicums will be attended by 73+23=96 students, except for patients' treatment, in which 73 students participate. For fifth year's patients' treatment 74+23=97 students have to be allotted.

At the end of August 1978 the final examinations of the academic year 1977-1978 were scheduled, determining next year's group for each student. As the timetables had to be issued at the start of the new academic year, a good two weeks were available for their construction. Because of the probationers, it was not possible to issue in September complete and final timetables for the whole year. In September therefore timetables were constructed for patients' treatment until Christmas and for the other practicums for the whole year. Only the first part of the timetables until Christmas was issued. In January patients' treatment for the remaining months of the academic year was assigned and the other practicums were adjusted if necessary; thereafter the second part of the timetables was issued to the students.

1.2. Approach

The necessary FORTRAN programs were developed in the months July and August, after drawing up the data of the practicums in June. We decided not to develop an integral optimizing model solving the whole problem at the same time, on account of the data gathered, which corroborated our initial

assessment that such a model should become too complicated and too expensive in use. In that case efficiency improvement should entail a tailoring of the model to the specific academic year, so as to interfere with an easy adaptation to organizing changes in later years.

The timetabling problem can be solved by the following stepwise approach. In the *first step* the students are assigned to dental instructors (Section 2). From that we know on which half days the students could get allotted for patients' treatment given the presence pattern of their instructor. In the *second step* we determine how and when the students participate in all practicums except for patients' treatment, assigning students as much as possible on those half days on which their instructor is absent (Section 3). In the *third step* patients' treatment is allotted based on the preceding steps (Section 4). Given the availability of students and staff we try to utilize the capacity of patients' treatment as fully as possible, thereby allotting each student about the same number of half days.

In this approach we firstly trace down the *conflicts* between the activities. A conflict between two activities exists if they are scheduled on the same half day. By analyzing these conflicts one can determine in most cases beforehand such an order of assignment of the activities that during the assignment process all conflicts can be solved. Basis of the programs used in this process are a set of subroutines to store or modify decisions in the individual timetables and to retrieve data and former decisions (Section 5).

2. PATIENTS' TREATMENT: GROUPING

The first step in the construction of the timetables is the assignment of students to dental instructors, i.e. the partition of all students of a given year into a number of groups, equal to the number of instructors for that year. Because all patients' treatment of a student has to be performed under the supervision of the same dental instructor, he or she can only be allotted for patients' treatment when the instructor in question is present. A dental instructor can supervise at most eight students at the same time, because of the intensity of the supervision and because of the layout of the clinic, where eight dental units are set up in one area.

The resulting timetables should observe two types of requirements with respect to the patients' treatment, as mentioned in the preceding section: the capacity should be equally distributed among the students, minimizing the unused capacity. Table 3 shows that some dental instructors are more

TABLE 3. AVAILABILITY OF FOURTH YEAR'S STAFF

Dental instructor	Segment				
	MON.AM	TUE.AM	TUE.PM	WED.PM	total/week
1	x	x	x		3
2		x	x	x	3
3	x	x	x		3
4	x	x	x	x	4
5	x	x		x	3
6	x	x	x	x	4

times available than other ones. Besides students are not always available for patients' treatment, because some practicums are scheduled simultaneously with patients' treatment. The available capacity is fully occupied only if for each instructor at least eight students of his group are available whenever he is present. Out of the available students eight are allotted for patients' treatment and the remaining ones are off.

For this problem it is important to distinguish between the normal, "new" students and the probationers. The new students attend all practicums belonging to their year; the probationers attend patients' treatment of their previous year and the other practicums of the next year, at least until Christmas. Because students belonging to the same category (new or probationer) are unavailable for patients' treatment about the same number of times, the students have to be distributed per category to the instructors proportionally to the availability of those. In this way and in view also of the numbers of students and staff, it should be possible, after the assignment of the other practicums, to allot about the same number of half days to each student for patients' treatment.

The exposition above only holds for the period up to Christmas. The probationers who, after Christmas, are admitted to the next year of their study without any further restriction, have done, at that time, no corresponding patients' treatment and it is desirable that they catch up their arrears. The grouping after Christmas, necessarily different, takes account of this situation as follows. The number of patients' treatments to allot after Christmas simply results from the availability of staff. The number of patients' treatments already performed by students of the year in question is added to this number; division of the sum by the number of students yields the standard number of patients' treatments for each student, setting aside for the moment students' availability. For each student now a target number is computed by subtracting from the standard the number of patients' treatments, already performed by him for that year. For a probationer the target number therefore equals the standard number. After this computation the groups can be composed such that the sum of the target numbers of a group is proportional to the number of times that the corresponding instructor will be present.

Mathematical model

In the period from September until Christmas each category of students (new or probationer) of each year has to be divided into groups in such a way that the size of the group is proportional to the availability of the dental instructors. This problem also occurs at the allocation of seats in

representative bodies based on proportional representation. The seats have to be divided into groups in such a way that the sizes of the groups are proportional to the numbers of votes cast for the various parties. The grouping problem of patients' treatment can be seen as this *proportional representation problem* by interpreting each student as a seat and each dental instructor as a party. The number of times that an instructor is present then is the number of votes cast for that party.

Both Balinski & Young [1] and Te Riele [3] describe several methods for the allocation of seats. Nearly all methods have advantages and disadvantages, e.g. favoring large or small parties. In our case methods favoring large parties are preferable: if a certain degree of unused capacity is inevitable, because of the conflicts with other practicums, the unused capacity of large groups will be proportionally less than of small groups, and hence the overrepresentation of the large groups will be counterbalanced.

In our approach we first constructed the grouping for patients' treatment and afterwards the assignment of the other practicums. We preferred this order above the reverse one because that order gives rise to a difficult mathematical programming problem. The latter problem is formulated in the Appendix.

3. ALLOCATION OF OTHER PRACTICUMS

3.1. Introduction

This section describes the organizational aspects of the dental practicums during the academic year 1978-1979, except for patients' treatment. For the construction of the timetables mainly the following aspects are essential:

- the period in which the practicum is scheduled and the half days within that period on which it can or must occur;
- the number of students which should attend simultaneously at least and/or at most;
- the pattern according to which the practicum must be attended, e.g. once a week on a fixed segment of the week or on ten consecutive mornings;
- the number of times for each student;
- participation by probationers yes or no;
- relations with other practicums, e.g. in the same week, before or after another practicum.

For each year's course (fourth, fifth, sixth) a separate program has been written to determine for each student when he should attend the practicums. In each program the practicums are processed in a properly chosen order; for each practicum is decided for each student on which half days he attends that practicum. There is no attempt to solve conflicts by reconsidering former decisions; in fact the method is a *single-pass* one. If it is impossible to allot a student to a practicum in a feasible way, the program produces a *message*. On account of those messages the program can be adapted; in practice those adaptations usually meant a simple change of the order, in which the students are allotted to a specific practicum. In all our runs we found in this manner at the second or third try a solution fulfilling all requirements.

3.2. The practicums

We describe in this section only the fourth year's practicums in the academic year 1978-1979; in other years similar practicums and comparable

problems occur.

TABLE 4. FOURTH YEAR'S ENDODONTICS

Segment	Date			
MON.AM	-	-	10:02*	10:09
MON.PM	-	-	-	-
TUE.AM	-	09:26	10:03	10:10
TUE.PM	-	09:26	10:03	10:10
WED.AM	-	-	-	-
WED.PM	-	09:27	10:04	10:11
THU.AM	-	-	-	-
THU.PM	-	-	-	-
FRI.AM	-	09:29	10:06	10:13
FRI.PM	09:22	09:29	10:06	10:13

The fourth year's practicum *endodontics* is trivially solvable: it takes place on 18 half days (Table 4) and all fourth year students should attend each time. Also the practicums *pathological anatomy, general activities, orthodontics* and *pharmacology* have to be attended by all students each time they are scheduled.

TABLE 5. FOURTH YEAR'S MICROBIOLOGY

Segment	Date				
MON.AM	-	10:23	10:30	11:06	11:13
TUE.AM	-	10:24	10:31	11:07	11:14
WED.PM	10:18	10:25	11:01	11:08	-
THU.PM	10:19	10:26	11:02	11:09	-
FRI.PM	10:20	10:27	11:03	11:10	-

Table 5 lists when *microbiology* is scheduled. The students have to be divided into two groups of about the same size; one group should

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* 10:02 = October 2nd.

attend the first ten half days of the practicum, the other one the last ten half days.

TABLE 6. FOURTH YEAR'S PROSTHODONTICS

Group	Segment	Period	
		from	until
1	TUE.PM	10:16	12:22
2	TUE.PM	01:29	03:30
3	TUE.PM	04:30	06:29

Prosthodontics is organized along similar lines, however the students have to be divided into three groups of about the same size (Table 6). Each student has to attend nine times. The period of group one contains ten Tuesdays; on 12:05 the practicum is canceled (Santa Claus).

Oral surgery allows participation of at most six students simultaneously. The practicum is scheduled from 11:20* until 06:29, interrupted by Christmas and Easter holidays and the examination periods. Each student has to attend every morning during one week. These requirements induce a division of the students into at least n/6 groups, with n the number of fourth year students. Because each group has to attend during a whole week, the number of groups cannot exceed the number of weeks that the practicum is being held.

For *dental materials* the students have to be divided into three groups of about the same size. The practicum consists of eight different parts and is being held 24 times. Because of the setup of instruments all students first have to attend the first part, next all the second part, and so on. Table 7 shows course of things of this practicum; the three groups are denoted by A, B and C.

Pedodontics is scheduled from 01:29 until 06:20, interrupted by the Easter break, and in this period on five segments of the week, MON.AM, TUE.AM, TUE.PM, WED.PM and FRI.AM. The students have to be divided into five groups; each group attends on a fixed segment of the week. So group one always on MON.AM, group two on TUE.AM and so on.

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*11:20 = November 20th.

TABLE 7. FOURTH YEAR'S DENTAL MATERIALS

Segment	Week starting on		
	01:08	01:15	01:22
MON.AM	A		A
MON.PM	B	B	B
TUE.AM	C		C
TUE.PM	A	C	A
WED.AM	B		B
WED.PM	C	A	C
THU.AM	A		A
THU.PM	B	B	
FRI.AM	C		B
FRI.PM	A	C	C

Roentgen diagnostics is scheduled from 11:20 until 06:29, interrupted by holidays and examination periods, every week on MON.AM and TUE.AM. At most four students can attend simultaneously. Each student has to attend two times, first all students for the first time and next all students for the second time. In this case there is no question of a division of students into fixed groups: four students attending together for the first time do not need to attend together for the second time. The whole period can be split up into two halfs; each student has to attend once in the first half and once in the second half.

Periodontics is scheduled from 01:29 until 03:09, except for 03:02, on FRI.PM. The only requirement is that each student attends twice.

Oral diagnosis is scheduled from 11:20 until 06:29 on WED.PM, interrupted by the Christmas and Easter break. Each student has to attend four consecutive times. Besides it is required that each time at most four students start; so from the fourth time on at most sixteen students are present. The resulting schedule is given in Table 8; the letters a,b,c,... denote a group of at most four students.

Oral hygiene is scheduled from 10:23 until 11:17 on WED.PM. The only requirement is that each student gets assigned the same number of times.

TABLE 8. FOURTH YEAR'S ORAL DIAGNOSIS

Date	Students			
11:22				a
11:29			a	b
12:06		a	b	c
12:13	a	b	c	d
12:20	b	c	d	e
01:10	c	d	e	f
01:17	d	e	f	g
01:24	e	f	g	h
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

Other activities do not need to take account of this practicum: all have priority on this practicum, which only is being held for the time left.

3.3. Conflicts between practicums

All scheduling problems arise from the fundamental fact that any student can participate in atmost one activity on a morning or afternoon. Therefore it is relevant to check which activities are scheduled simultaneously and how these conflicts can be resolved.

The practicums endondontics, pathological anatomy, general activities, orthodontics and pharmacology cause mutually and with the other activities no conflicts. In the period of microbiology the only other practicum scheduled is prosthodontics. Because the latter is being held solely on TUE.PM and the former never on that segment, no conflict arises.

On FRI.PM there is no other practicum than periodontics which consequently causes no conflicts.

For the other practicums Table 9 shows in which period and on which segments they are scheduled, at least so far as conflicts could arise from it. Dental materials which shares segments with all other practicums of Table 9, has only a conflict with roentgen diagnostics and oral diagnosis,

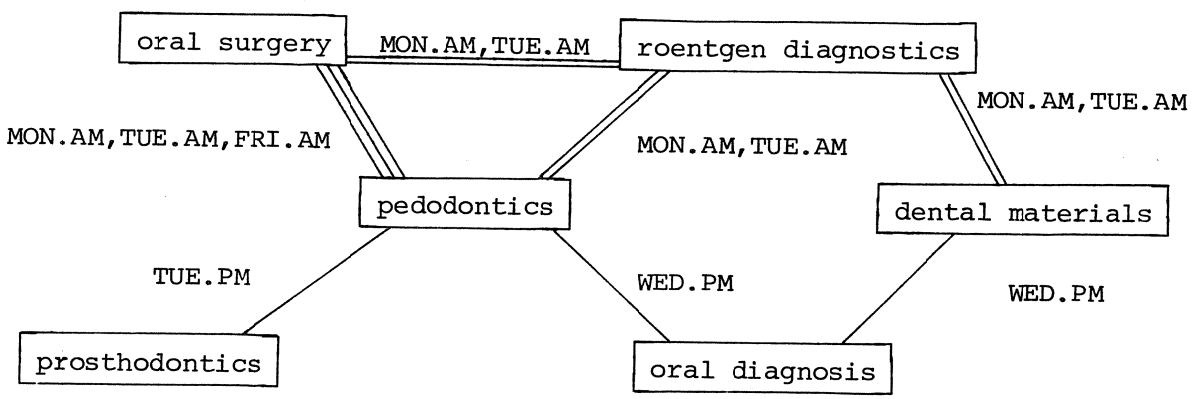
TABLE 9. CONFLICTS FOURTH YEAR'S PRACTICUMS

Practicum	Segment					Period	
	MON.AM	TUE.AM	TUE.PM	WED.PM	FRI.AM	from	until
prosthodontics			x			10:16	06:29
oral surgery	x	x			x	11:20	06:29
pedodontics	x	x	x	x	x	01:29	06:29
roentgen diagnostics	x	x				11:20	06:29
oral diagnosis				x		11:20	06:29
dental materials	x	x	x	x	x	01:08	01:26

because the other practicums are interrupted from 01:08 until 01:26.

Prosthodontics is scheduled only on TUE.PM; hence students allocated to the TUE.PM group of pedodontics could be unable to attend prosthodontics. Pedodontics starts on 01:29, prosthodontics on 10:16. By assigning all students of the TUE.PM group of pedodontics for prosthodontics in the period from 10:16 until 12:22 this conflict can be resolved. This simple solution is feasible because students have to be divided for pedodontics into five groups and for prosthodontics into three, hence larger groups.

FIGURE 1. CONFLICTS FOURTH YEAR'S PRACTICUMS



Along the same lines the conflict between oral diagnosis and pedodontics on WED.PM can be resolved. In the period from 11:20 until 01:26 twenty

students, i.e. more than one fifth of the total number of fourth year students (groups a - e in Table 8) can complete oral diagnosis; after that they can attend pedodontics on WED.PM.

The conflict between oral surgery and pedodontics really is unsolvable. We decided to rank oral surgery above pedodontics: during the week that a student attends oral surgery, he is allowed to skip pedodontics. The number of students which miss one half day of pedodontics can be reduced by assigning only students of the MON.AM, TUE.AM and FRI.AM groups of pedodontics before Christmas to oral surgery.

To solve the conflicts between roentgen diagnostics on the one side and oral surgery and pedodontics on the other side it is sufficient to allot first the latter two activities and after that roentgen diagnostics. In such a way the students are assigned for roentgen outside the week in which they attend oral surgery, and on a different half day than for pedodontics.

The same type of solution is feasible for the conflicts concerning dental materials. This practicum which is scheduled during three weeks in January, is assigned first; next, the students are assigned for oral diagnosis and roentgen diagnostics on half days, on which they do not attend dental materials, what easily can be done because of the latitude for the latter activities.

4. PATIENTS' TREATMENT: ALLOCATION

4.1. Approach

The third step in the construction of timetables consists of the allocation to each student of the half days on which he is doing patients' treatment.

On account of the grouping found in the first step we know for each student when his instructor is available. On account of the allotment to the other practicums found in the second step we know for each student when he himself is available. On each half day that a dental instructor is present, at most eight students have to be chosen from the on that time available students of his group. Availability of less than eight students implies that part of the capacity of patients' treatment remains unused. The solutions found in practice showed an unused capacity of 1 to 2 percent, which was very acceptable.

The allocation of students can not decrease the unused capacity, of course. Therefore the allocation is solely governed by the desirability that all students are doing about the same number of patients' treatments. Given the partition of students into groups the allocation can be performed for each group separately. In practice only small differences between the groups occurred.

Within a group, allocations can be made successively for the half days during the year on which the group's instructor is present. On a given half day therefore the assignments at former occasions are known. For the new allocation each student of the group is awarded a score, equal to the number of previous assignments of the student plus the expected number of future assignments if the allocation from now on should be determined by a random draw out of the available students. The eight available students with the lowest scores are then assigned.

4.2. Mathematical model

Given the grouping of patients' treatment and the allotments to the other practicums the allocation to patients' treatment can be executed separately for each group.

Suppose a group consists of n students and its instructor is present on half days 1,2,...,m. Coefficient v_{ih} denotes whether student i is available on half day h ($v_{ih} = 1$) or not ($v_{ih} = 0$). An instructor can supervise simultaneously at most c students. Each student should get assigned at least m^- and at most m^+ patients' treatments. The variables of the problem are x_{ih}, with $x_{ih} = 1$ if student i is doing patient treatment on half day h, and $x_{ih} = 0$ else. We now formulate the allocation problem as follows;

$$\text{Maximize} \sum_{i=1}^{n} \sum_{h=1}^{m} x_{ih} \tag{1}$$

subject to

$$\sum_{i=1}^{n} x_{ih} \leq c \qquad (h = 1,\ldots,m) \tag{2}$$

$$m^- \leq \sum_{h=1}^{m} x_{ih} \leq m^+ \qquad (i = 1,\ldots,n) \tag{3}$$

$$x_{ih} \leq v_{ih} \qquad (i = 1,\ldots,n;\ h = 1,\ldots,m) \tag{4}$$

$$x_{ih} \in \{0,1\} \qquad (i = 1,\ldots,n;\ h = 1,\ldots,m) \tag{5}$$

It is no simple matter to select a priori the correct value of m^-. In fact we have to determine the largest m^- such that constraints (2) - (5) have a feasible solution, and for that value of m^- the object function (1), the total number of patients' treatments of the group, should be maximized. Problem (1) - (5) is a generalized linear assignment problem with upper-bounds; solution methods can be found in Ross & Soland [4].

In our timetabling problem we used the following heuristic for this problem. Let allocations for half days 1,2,...,t-1 have already been made, so x_{ih} is known for $h < t$ and all i. Suppose that for the remaining half days t,t+1,...,m the assignment is determined by a random draw out of the available students. The probability of assigning student i on half day h ($h \geq t$) is

$$p_{ih} = \min\{v_{ih}, c/\sum_{j=1}^{n} v_{jh}\}.$$

Therefore, on half day t, the expectation of the total number of patients' treatments of student i is equal to

$$e_{it} = \sum_{h=1}^{t-1} x_{ih} + \sum_{h=t}^{m} p_{ih}.$$

An obvious rule for the assignment on time t is to assign the available students in order of increasing e_{it}. By executing this procedure for $t = 1,\ldots,m$ we found in practice an equally distributed allocation to patients' treatment.

5. DATA AND PROGRAMS

5.1. Input data

The programs use for input general data, student data, staff data and practicum data.

The *general data* include calendar data and data about the numbers of students.

In the academic year 78-79 the calendar data consisted of the number 78 (to print the academic year in the output), the number of days in each month of the year (because of possible lapse years) and the number 1809 as first date of the academic year. By this number also the relation between date and day of the week is given for all days of the year, because the first date must be a Monday. Internally the days of the year are consecutively indexed from 1 up to 365; in in- and output the usual notation (25 FEB 79) is being used.

The numbers of students are recorded by means of six numbers: for each year (fourth, fifth and sixth) and for each category (new or probationer) the number.

The *student data* contain for each student his surname and initials, university registration number, year (4, 5 or 6) and a letter denoting his category.

The student data are ordered by student year; within a year first new students, next probationers, and within a category alphabetically. By this ordering the student data are conveniently sequenced both for patients' treatment as for the other practicums. E.g. the fifth year's probationers are doing patients' treatment until Christmas together with the new fifth year's students and attend the other practicums together with the new sixth year's students.

The *staff data* contain for each dental instructor his name, the year which he is supervising (4, 5 or 6) and his weekly pattern of availability. Example of a weekly pattern: MON.AM TUE.PM FRI.AM.

The *practicum data* contain for each practicum an index number, the name (maximal eight characters) which denotes the activity in the output, the year to which the practicum belongs (4, 5 or 6), and one or more periods in which it is scheduled. For each period the first date (a Monday), the last date (a Friday) and the weekly pattern of the period.
Example: 9PERIO 5 02260323 WED.PM.
Besides the proper activities a dummy practicum called "NOPRAC" has been introduced to allow for holidays.

We see in the foregoing that some data are contained in the input in several ways. This redundancy is functional and enables a check of the correctness of the input data. Such checks surely serve a purpose in view of the fact that some data are transmitted by telephone or are changed at the very last moment.

5.2. Programs

A set of FORTRAN programs has been written to solve the timetabling problem according to the three-step approach described above. Basis of the programs are a set of subroutines performing administrative duties. These subroutines are expected to be independent of the organizational framework of the dental school and so to allow for a large degree of flexibility in case of changes of the courses.

The basis contains routines to read the input data, to present the results, to store and to retrieve (intermediate) results, to initialize the timetables and to modify timetables of individual students or a group of students. All these subroutines are safeguarded against erroneous changes of the timetables. Moreover a system of messages has been provided, warning the user in case the program leaves any student unassigned for any practicum or tries erroneous changes.

Within the programs a hierarchical structure exists. The programs dealing with the grouping and the allocation of patients' treatment are applicable for each of the three years. For each year a separate program deals with the allocation of the other practicums, incorporating their organizational aspects. When those aspects change the program in question has

to be modified. According to our experience during two years this can be done rather easily thanks to the available basic routines. All programs are called from one main program and can be used interactively due to an overlay structure.

The storage of the individual timetables has been arranged as follows. The academic year consists of 52 weeks and each week of 10 segments. An individual timetable henceforth contains 520 data, viz. the numbers of the successive activities of the student. These data are stored into 52 machine words, one word of 60 bits a week. For each segment 6 bits are available for the number of the activity, therefore 64 activities can be distinguished.

We did not opt for such a design of the programs that the staff of the dental school could handle them independently. The staff has been involved in judging the produced timetables, in consequence of which the programs were slightly modified, if necessary, to generate better timetables. In all cases acceptable timetables have been found after a couple of tries. The type and location of those for the rest straightforward modifications was always to such an extent dependent on the specific situation as to make it too expensive and even infeasible to strive for a framework of the programs in which those modifications could be specified by means of keywords or codes.

5.3. Output data

The most important part of the output consists of the individual timetables of the students. For each half day is indicated in which activity the student should participate.

Besides a survey is produced for each dental instructor and for each of the other practicums, which records for each half day which students are assigned for that half day to that activity.

Furthermore a survey is produced for each half day of the activities of all students. By means of this survey management easily can trace down the whereabouts of a student on a given time.

Finally various statistical reports with respect to the participation

in activities and the utilization of the capacity for patients' treatment are being prepared.

APPENDIX: A MATHEMATICAL MODEL FOR THE GROUPING OF PATIENTS' TREATMENT

This appendix describes a mathematical model for the grouping of students for patients' treatment. The model is applicable in an approach which first assigns the other practicums and afterwards constructs the groups for patients' treatment.

Suppose n students have to be partitioned into p groups, each corresponding with a dental instructor. The p instructors are available on half days 1,...,m. On account of the allotment of the other practicums we know when a student is available for patients' treatment: we define coefficients $v_{ih} = 1$, if student i is available on half day h, else $v_{ih} = 0$. Each instructor can supervise simultaneously at most c students. The students are partitioned into p groups, with a size varying between b_g^- and b_g^+ (g = 1,...,p). These bounds can be chosen taking into account staff's availability. The variables of the problem are x_{ig} and y_{gh}. The value of x_{ig} denotes whether student i has been assigned to group g ($x_{ig} = 1$) or not ($x_{ig} = 0$). The number of patients' treatments on half day h by students of group g is defined as y_{gh}. We now formulate the grouping problem as follows.

$$\text{Maximize} \sum_{g=1}^{p} \sum_{h=1}^{m} y_{gh} \tag{6}$$

subject to

$$\sum_{g=1}^{p} x_{ig} = 1 \qquad (i = 1,\ldots,n) \tag{7}$$

$$b_g^- \leq \sum_{i=1}^{n} x_{ig} \leq b_g^+ \qquad (g = 1,\ldots,p) \tag{8}$$

$$y_{gh} \leq \sum_{i=1}^{n} v_{ih} x_{ig} \qquad (g = 1,\ldots,p;\ h = 1,\ldots,m) \tag{9}$$

$$y_{gh} \leq c \qquad (g = 1,\ldots,p;\ h = 1,\ldots,m) \tag{10}$$

$$x_{ig} \in \{0,1\} \qquad (i = 1,\ldots,n;\ g = 1,\ldots,p) \tag{11}$$

$$y_{gh} \geq 0 \qquad (g = 1,\ldots,p;\ h = 1,\ldots,m) \tag{12}$$

The object function (6) maximizes the total number of patients' treatments. Constraints (7) imply that each student is assigned to exactly one

group. Constraints (8) bound the sizes of the groups. The number of students of a group, assigned for patients' treatment, is at any moment at most equal to the number of available students of the group, because of (9), and never exceeds the capacity of the instructor, because of (10).

In the above given model we suppose that each dental instructor is available on each half day. To incorporate absence of staff we should replace (10) by $y_{gh} \leq ca_{gh}$, with a_{gh} denoting presence ($a_{gh} = 1$) or absence ($a_{gh} = 0$) of instructor g on half day h.

Problem (6)-(12) is a nonlinear generalized assignment problem. It can be reformulated as

$$\text{Maximize} \sum_{g=1}^{p} \sum_{h=1}^{m} \min\{c, \sum_{i=1}^{n} v_{ih} x_{ig}\}$$

subject to (7), (8) and (11).

The problem could be solved also via Lagrangian relaxation: by relaxing constraints (9) and (10) a linear generalized assignment problem arises.

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